International Journal of Theoretical Physics, Vol. 40, No. 5, 2001

Fermionic Entropy in Kerr Black Hole Space–Time Background

You-Gen Shen[1,2,3,4] and Zong-Yi Cheng[1]

Received January 18, 2000

Making use of brick-wall model proposed by 't Hooft, we have obtained the free energy and the entropy of Fermionic field and given out their expressions under the Kerr space–time background.

1. INTRODUCTION

In theoretical physics, black hole thermodynamics is an enigma. It is also a junction of general relativity theory, quantum mechanics, and statistics physics.

Since Bekenstein and Hawking proposed in the 1970s that black hole entropy is proportional to its horizon area (Bekenstein, 1972, 1973, 1974; Hawking, 1975; Kallosh *et al.*, 1993), people have been exploring the statistical origin of black hole entropy. One of the methods most used to study the statistical origin of black hole entropy is the brick-wall method proposed by 'tHooft (1985). Making use of this method, 'tHooft investigated the free scale field's statistical characteristic in Schwarzshild black hole background, obtained an expression of entropy in terms of the horizon area, and verified that the entropy is proportional to its horizon area. What is more, the entropy can be written as $S = \frac{A_h}{4}$ when the cutoff factor satisfies a certain condition. When the cutoff factor approaches zero, the entropy is divergent. He thought that this kind of divergence is caused by going to infinite state density at the vicinity of the horizon.

[1] Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, People's Republic of China.
[2] National Astronomical Observatory, Chinese Academy of Sciences, Beijing 100012, People's Republic of China.
[3] Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, People's Republic of China.
[4] To whom correspondence should be addressed at Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, People's Republic of China; e-mail: ygshen@center.shao.ac.cn

Another method to study the statistical origin of black hole entropy, which is actually equivalent (Callan and Wilczek, 1994; Kabat and Strassler, 1994) to the brick-wall model, was adopted by Bombell *et al.* (1986) and Srednicki (1993). Starting with the one-loop action of a scalar massive field and making use of Euclidean path integration of Gibbons and Hawking (1977), Solodukhin (1995a,b) studied the quantum corrections to the black hole entropy. In quantum mechanics, the geometrical entropy satisfies following assumptions: the entropy will be called Boson entropy if the particle is a Boson that obeys Bose–Einstein satatistical distrubution; the entropy will be called Fermionic entropy if geometrical entropy in quamtum mechanics is calculated by means of counting Fermi state.

Since the mid of 1990s, many researchers have been interested in the questions of black hole entropy (Brown, 1995; Carlip and Teitelboim, 1995; Carlip, 1995; Cognola and Lecca, 1998; Cvetic and Youn, 1996; de Alwis and Ohta, 1995; Demers *et al.*, 1995; File *et al.*, 1994; Ghosh and Mitra, 1994, 1995; Gubser *et al.*, 1996; Hawking *et al.*, 1995; Ichinose and Satoh, 1995; iLarsen and Wilczek, 1996; Jacobson *et al.*, 1995; Kabat *et al.*, 1995; Kim *et al.*, 1997; Larsen and Wilczek, 1995; Lee and Kim, 1996; Lee and Kim, 1996; Lee *et al.*, 1996; Lee *et al.*, 1996; Mann and Solodukhin, 1996; Pinto and Soares, 1995; Russo, 1995; Shen and Chen, 1998, 1999a,b, 1999; Shen *et al.*, 1997; Solodukhin, 1995; Solodukhin, 1996; Solodukhim, 1995a,b; Susskind and Uglum, 1994; Teitelboin, 1995; Zhou *et al.*, 1995). But up to now, entropy of free scale field was mainly studied by the people; rather few studied the entropy of Dirac's spinor field; there is only a few research for its lower dimenssion field. So it is nescessary to further study the entropy of a four-dimensional Dirac field.

In this paper, making use of 'tHooft's brick-wall model in four-dimensional Dirac spinor field, we obtained the free energy and the entropy of Fermionic field, and gave their expressions in the Kerr black hole space-time background.

2. DIRAC FIELD EQUATIONS

In curved space-time, the spinor representations of massless Dirac equations can be expressed as Teukolsky (1973)

$$\nabla_{A\dot{B}} P^A = 0, \tag{1}$$

$$\nabla_{A\dot{B}} Q^A = 0, \tag{2}$$

where P^A and Q^A are 2 two-components spinors; ∇_{AB} is the spinor covariant differentiation; $\nabla_{A\dot{B}} = \sigma^{\mu}_{A\dot{B}} \nabla_{\mu}$, $\sigma^{\mu}_{A\dot{B}}$ are 2×2 Hermitian matrices; they satisfy $g_{\mu\nu}\sigma^{\mu}_{A\dot{B}}\sigma^{\mu}_{C\dot{D}} = \epsilon_{AC}\epsilon_{\dot{B}\dot{D}}$; ϵ_{AC} and $\epsilon_{\dot{B}\dot{D}}$ are the anti-symmetric Levi–Civita symbols, ∇_{μ} is covariant differentiation.

It is well known that the metric for Kerr black hole is Kerr (1963)

$$ds^2 = \left[1 - \frac{2Mr}{\Sigma}\right] dt^2 - \frac{\Sigma}{\Delta_r}\, dr^2 - \Sigma\, d\theta^2 - \left[\frac{2Mra \sin^2\theta}{\Sigma} + (r^2 + a^2)\right] \times \sin^2\theta\, d\varphi^2 + \frac{4Mra}{\Sigma} \sin^2\theta\, dt\, d\varphi, \tag{3}$$

where

$$\Sigma = r^2 + a^2 \cos^2\theta,$$
$$\Delta_r = r^2 + a^2 - 2Mr,$$
$$a = \frac{J}{M},$$

where M and a are the mass of Kerr black hole and angular momentum per unit mass respectively.

Choose the null tetrad as follows Chandrasekhar (1963):

$$\begin{aligned} l^\mu &= \frac{1}{\Delta_r}(r^2 + a^2, \Delta_r, 0, a), \\ n^\mu &= \frac{1}{2\Sigma}(r^2 + a^2, -\Delta_r, 0, a), \\ m^\mu &= \frac{1}{\sqrt{2}\bar{\rho}}\left(ia \sin\theta, 0, 1, \frac{i}{\sin\theta}\right), \\ \bar{m}^\mu &= \frac{1}{\sqrt{2}\bar{\rho}^\star}\left(-ia \sin\theta, 0, 1, -\frac{i}{\sin\theta}\right), \end{aligned} \tag{4}$$

where $\bar{\rho} = r + ia\cos\theta$, $\bar{p}^\star = r - ia\cos\theta$. It is easily seen that the tetrad in (4) is a null vector, that is,

$$l_\mu l^\mu = n_\mu n^\mu = m_\mu m^\mu = \bar{m}_\mu \bar{m}^\mu = 0, \tag{5}$$

satisfies a pseudo-orthogonality relationship, that is,

$$\begin{aligned} l_\mu n^\mu &= -m_\mu \bar{m}^\mu = 1, \\ l_\mu m^\mu &= l_\mu \bar{m}^\mu = n_\mu m^\mu = n_\mu \bar{m}^\mu = 0, \end{aligned} \tag{6}$$

and satisfies metric conditions, that is,

$$g_{\mu\nu} = l_\mu n_\nu + n_\mu l_\nu - m_\mu \bar{m}_\nu - \bar{m}_\mu m_\nu. \tag{7}$$

Suppose spinor bases $\zeta_a^A = \delta_a^A$; here, A is the index of spinor component and a is the index of spinor base; A and a take the values 0 or 1.

The covariant differentiation $\nabla_{A\dot{B}}\xi^A$ for an arbitray spinor ξ^A can be expressed as the component along the direction of spinor base ζ_a^A, that is,

$$\zeta_a^A \zeta_b^B \zeta_C^c \nabla_{A\dot{B}}\xi^C = \nabla_{a\dot{b}}\xi^c = \partial_{a\dot{b}}\xi^c + \Gamma^c_{da\dot{b}}\xi^d, \tag{8}$$

where $\partial_{a\dot{b}}$ is usual spinor differentiation, and $\Gamma^c_{da\dot{b}}$ is the spinor coefficient.

Let

$$\begin{aligned}
\partial_{0\dot{0}} &= l^\mu \partial_\mu \equiv D,\\
\partial_{1\dot{1}} &= n^\mu \partial_\mu \equiv \Delta,\\
\partial_{0\dot{1}} &= m^\mu \partial_\mu \equiv \delta,\\
\partial_{1\dot{0}} &= \bar{m}^\mu \partial_\mu \equiv \bar{\delta},
\end{aligned} \tag{9}$$

the Dirac Eqs. (1) and (2) can be reduced to four coupled equations as follows:

$$\begin{aligned}
&(D + \varepsilon - \rho)F_1 + (\bar{\delta} + \pi - \alpha)F_2 = 0,\\
&(\Delta + \mu - \gamma)F_2 + (\delta + \beta - \tau)F_1 = 0,\\
&(D + \varepsilon^\star - \rho^\star)G_2 - (\delta + \pi^\star - \alpha^\star)G_1 = 0,\\
&(\Delta + \mu^\star - \gamma^\star)G_1 - (\bar{\delta} + \beta^\star - \gamma^\star)G_2 = 0,
\end{aligned} \tag{10}$$

where F_1, F_2, G_1, and G_2 are the spinor quantities of which there are four components, among them, $F_1 = P^0$, $F_2 = P^1$, $G_1 = \bar{Q}^{\dot{1}}$, and $G_2 = -\bar{Q}^{\dot{0}}$; α, β, γ, ϵ, μ, π, ρ, and τ, etc., are the Newman–Penrose symbols Newman and Penrose (1962), and $\alpha^\star$, $\beta^\star$, etc., are the conjugates of the α, β, etc., and the relationship between them and the null tetrad is

$$\begin{aligned}
\alpha &= \frac{1}{2}(l_{\mu;\nu}n^\mu \bar{m}^\nu - m_{\mu;\nu}\bar{m}^\mu \bar{m}^\nu),\\
\beta &= \frac{1}{2}(l_{\mu;\nu}n^\mu m^\nu - m_{\mu;\nu}\bar{m}^\mu m^\nu),\\
\gamma &= \frac{1}{2}(l_{\mu;\nu}n^\mu n^\nu - m_{\mu;\nu}\bar{m}^\mu n^\nu),\\
\varepsilon &= \frac{1}{2}(l_{\mu;\nu}n^\mu l^\nu - m_{\mu;\nu}\bar{m}^\mu l^\nu),\\
\pi &= -n_{\mu;\nu}\bar{m}^\mu \bar{m}^\nu,\\
\rho &= l_{\mu;\nu}m^\mu \bar{m}^\nu,\\
\tau &= l_{\mu;\nu}m^\mu n^\nu.
\end{aligned} \tag{11}$$

After a tedious but straightforward calculation, we have Chandrasekhar (1963)

$$\rho = -\frac{1}{\rho^\star},$$

$$\beta = \frac{\cot\theta}{2\sqrt{2}\bar{\rho}},$$
$$\pi = \frac{ia\sin\theta}{\sqrt{2}(\rho^{\star})^2},$$
$$\tau = -\frac{ia\sin\theta}{\sqrt{2}\Sigma},$$
$$\mu = -\frac{\Delta_r}{2\Sigma\bar{\rho}^{\star}},$$
$$\gamma = \mu + \frac{r-M}{2\Sigma},$$
$$\alpha = \pi - \beta^{\star},$$
$$\varepsilon = 0. \tag{12}$$

Let

$$F_1 = e^{-i\omega t + im\varphi} f_1(r,\theta),$$
$$F_2 = e^{-i\omega t + im\varphi} f_2(r,\theta),$$
$$G_1 = e^{-i\omega t + im\varphi} g_1(r,\theta),$$
$$G_2 = e^{-i\omega t + im\varphi} g_2(r,\theta). \tag{13}$$

Then (10) becomes

$$\left(\mathcal{D}_0 + \frac{1}{\bar{\rho}^{\star}}\right) f_1 + \frac{1}{\sqrt{2}\bar{\rho}^{\star}}\mathcal{L}_{1/2} f_2 = 0,$$
$$\frac{\Delta_r}{2\Sigma}\mathcal{D}^{+}_{1/2} f_2 - \frac{1}{\sqrt{2}\bar{\rho}}\left(\mathcal{L}_1 + \frac{1}{\bar{\rho}^{\star}} ia\sin\theta\right) f_1 = 0,$$
$$\left(\mathcal{D}_0 + \frac{1}{\bar{\rho}}\right) g_2 - \frac{1}{\sqrt{2}\bar{\rho}}\mathcal{L}^{+}_{1/2} g_1 = 0,$$
$$\frac{\Delta_r}{2\Sigma}\mathcal{D}^{+}_{1/2} g_1 + \frac{1}{\sqrt{2}\bar{\rho}^{\star}}\left(\mathcal{L}_{1/2} - \frac{1}{\bar{\rho}} ia\sin\theta\right) g_2 = 0, \tag{14}$$

where

$$\mathcal{D}_n = \partial_r - \frac{ik}{\Delta_r} + 2n\frac{r-M}{\Delta_r},$$
$$\mathcal{D}^{+}_n = \partial_r + \frac{ik}{\Delta_r} + 2n\frac{r-M}{\Delta_r},$$
$$\mathcal{L}_n = \partial_\theta - H + \cot\theta,$$

$$\begin{aligned}\mathcal{L}_n^+ &= \partial_\theta + H + n\cot\theta,\\ K &= (r^2+a^2)\omega - am,\\ H &= a\omega\sin\theta - \frac{m}{\sin\theta}.\end{aligned} \tag{15}$$

Making the following transformation

$$\begin{aligned}U_1(r,\theta) &= \bar{\rho}^\star f_1(r,\theta),\\ U_2(r,\theta) &= f_2(r,\theta),\\ V_1(r,\theta) &= g_1(r,\theta),\\ V_2(r,\theta) &= \bar{\rho} g_2(r,\theta),\end{aligned} \tag{16}$$

(14) becomes

$$\begin{aligned}\mathcal{D}_0 U_1 + \frac{1}{\sqrt{2}}\mathcal{L}_{1/2} U_2 &= 0,\\ \Delta_r \mathcal{D}_{1/2}^+ U_2 - \sqrt{2}\mathcal{L}_{1/2}^+ U_2 &= 0,\\ \mathcal{D}_0 V_2 - \frac{1}{\sqrt{2}}\mathcal{L}_{1/2}^+ V_1 &= 0,\\ \Delta_r \mathcal{D}_{1/2}^+ V_1 + \sqrt{2}\mathcal{L}_{1/2} V_2 &= 0.\end{aligned} \tag{17}$$

Let

$$\begin{aligned}U_1(r,\theta) &= R_{-1/2}(r)S_{-1/2}(\theta),\\ U_2(r,\theta) &= R_{+1/2}(r)S_{+1/2}(\theta),\\ V_1(r,\theta) &= R_{+1/2}(r)S_{-1/2}(\theta),\\ V_2(r,\theta) &= R_{-1/2}(r)S_{+1/2}(\theta).\end{aligned} \tag{18}$$

Having substituted Eqs. (18) into Eqs. (17) and separated the variables, we have

$$\begin{aligned}\Delta_r \mathcal{D}_{1/2}^+ \mathcal{D}_0 R_{-1/2} - \lambda^2 R_{-1/2} &= 0,\\ \mathcal{D}_0 \Delta_r \mathcal{D}_{1/2}^+ R_{+1/2} - \lambda^2 R_{+1/2} &= 0,\\ \mathcal{L}_{1/2}^+ \mathcal{L}_{1/2} S_{+1/2} + \lambda^2 S_{+1/2} &= 0,\\ \mathcal{L}_{1/2}^+ \mathcal{L}_{1/2}^+ S_{-1/2} + \lambda^2 S_{-1/2} &= 0,\end{aligned} \tag{19}$$

where λ^2 is the constant of separating variables. Substitution of Eqs. (18) into

Eqs. (19) gives

$$\Delta_r \partial_r^2 R_{-1/2} + (r - M)\partial_r R_{-1/2} + \left[\frac{K^2}{\Delta_r} - 2i\omega r + iK\frac{r - M}{\Delta_r} - \lambda^2\right] R_{-1/2} = 0. \tag{20}$$

$$\Delta_r \partial_r^2 R_{+1/2} + 3(r - M)\partial_r R_{+1/2} + \left[\frac{K^2}{\Delta_r} + 2i\omega r - iK\frac{r - M}{\Delta_r} - \lambda^2 + 1\right] R_{+1/2} = 0. \tag{21}$$

$$\left[\frac{1}{\sin\theta}\,\partial_\theta \sin\theta\,\partial_\theta - \frac{m^2}{\sin^2\theta} + \lambda^2\right] S_{\pm 1/2} + \left[\frac{1}{4}\cot^2\theta \mp \frac{m\cos\theta}{\sin^2\theta}, - \frac{1}{2\sin^2\theta} \mp a\omega\cos\theta - a^2\omega^2\sin^2\theta + 2am\omega\right] S_{\pm 1/2} = 0. \tag{22}$$

3. THE FERMIONIC ENTROPY

Because the Dirac wave function has four components and the entropy is equal to the sum of the entropies of every component, to obtain the entropy of Dirac field, we have to calculate the entropy of its every component and then add them together. First, let us calculate the entropy of the F_1 component. Using brick-wall model, and assuming that the wave function vanishes near horizon within a range of h (h is a positive infinite decimal), that is,

$$F_1(r) = 0 \tag{23}$$

when $r \leq r_+ + h$, the wave function would also vanish away from horizon L, that is,

$$F_1(r) = 0 \tag{24}$$

when $r \geq L$. $r_+ = M + \sqrt{M^2 - a^2}$ is the event horizon of Kerr black hole, h is the ultraviolet cutoff factor, L is the ultrared cutoff factor, and $L \gg r_+$.

The radial component $R_{-1/2}$ of F_1 satisfies (20).

Let $R_{-1/2} = e^{i\omega_1(r)}$, based on the WKB approximation; it follows that

$$K_1^2 = \Delta_r^{-1}\left[\frac{K^2}{\Delta_r} - l(l + 1)\right]. \tag{25}$$

where $k_1 = \partial_r W_1(r)$, it is radial wave number.

Suppose that the studied Dirac field is in the vacuum state of Hartle–Hawking (Hartle and Hawking, 1996), and the temperature of Dirac field is the Hawking temperature $T_{\mathrm{H}} = \frac{K}{2\pi} = \frac{r_+ - r_-}{4\pi(r^2 + a^2)}$. According to the canonical ensemble theory, the free energy of Fermi system is:

$$\beta E_1 = -\Sigma\, ln(1 + e^{-\beta\omega}), \tag{26}$$

where β is the inverse of Hawking temperature. In a semiclassical treatment, the visible energy state is continuous distribution, and the sum can be changed to integration:

$$\sum_{\omega} \longrightarrow \int_0^{\infty} d\omega\, g(\omega), \tag{27}$$

where $g(\omega)$ is the state density, $g(\omega) = \frac{d\Gamma(\omega)}{d\omega}$, $\Gamma(\omega)$ is the microcosmic state number, that is,

$$\Gamma(\omega) = \sum_{l,m} n_r(\omega, l, m), \tag{28}$$

where n_r is a nonnegative integer number, and

$$n_r\pi = \int_{r_+ + h}^{L} K_1(r, l, m)\, dr, \tag{29}$$

Taking sum for angle quantum number is also treated as integration, and it is required that $K_1(r, l, m) \geq 0$ during integration process, and then we have

$$\Gamma(\omega) = \int (2l + 1)\, dl \cdot \frac{1}{\pi} \int K_1\, dr, \tag{30}$$

and the free energy can be expressed as

$$\beta E_1 = \frac{\beta}{\pi} \int dl\, (2l + 1) \int d\omega \left(1 + e^{\beta(\omega - m\Omega_0)}\right)^{-1} \cdot \int_{r_+ + h}^{L} K_1\, dr, \tag{31}$$

where $\Omega_0 = -\frac{g_{t\varphi}}{g_{\varphi\varphi}}|_{r=r_+}$ is the rotation angle velocity of Dirac field. It follows that

$$\begin{aligned}
E_1 = &-\frac{7\pi^3}{180h} \cdot \frac{1}{\beta^4} \cdot \frac{(r_+^2 + a^2)^3}{(r_+ - r_-)^2} + \frac{7\pi^3}{60h\beta^4} \cdot \frac{am(r_+^2 + a^2)^2}{(r_+ - r_-)^2} \\
&- \frac{7\pi^3}{60h\beta^4} \cdot \frac{a^2m^2(r_+^2 + a^2)}{(r_+ - r_-)^2} + \frac{7\pi^3}{180h\beta^4} \cdot \frac{a^2m^3}{(r_+ - r_-)^2} \\
&+ \frac{9}{16\pi^3} \cdot \frac{\zeta(3)}{h\beta} \cdot am^2\Omega_0 - \frac{9}{4\pi^2} \cdot \frac{\zeta(3)}{h\beta^2} \cdot a^2m^3\Omega_0 \cdot \frac{1}{r_+ - r_-} \\
&+ \frac{3}{\pi} \cdot \frac{\zeta(3)}{h\beta^3} \cdot a^3m^4\Omega_0 \cdot \frac{1}{(r_+ - r_-)^2} + \frac{\pi}{2h\beta^2} \cdot am^3\Omega_0^2 \cdot \frac{(r_+^2 + a^2)^2}{(r_+ - r_-)^2} \\
&- \frac{\pi}{2h\beta^2} \cdot a^2m^4\Omega_0^2 \cdot \frac{r_+^2 + a^2}{(r_+ - r_-)^2} + \frac{\pi}{6h\beta^2} \cdot a^3m^5\Omega_0^2 \cdot \frac{1}{(r_+ - r_-)^2}
\end{aligned}$$

$$+\frac{2}{\pi}\frac{1}{h\beta}\cdot\zeta(1)\cdot am^4\Omega_0^3\cdot\frac{(r_+^2+a^2)^2}{(r_+-r_-)^2}-\frac{2}{\pi}\frac{1}{h\beta}\cdot\zeta(1)\cdot a^2m^5\Omega_0^3\cdot\frac{r_+^2+a^2}{(r_++r_-)^2}$$
$$+\frac{2}{3\pi}\frac{1}{h\beta}\cdot\zeta(1)\cdot a^3m^6\Omega_0^3\cdot\frac{1}{(r_+-r_-)^2}-\frac{3}{\pi}\frac{1}{h\beta^3}\cdot\zeta(3)\cdot m\Omega_0\cdot\frac{(r_+^2+a^2)^3}{(r_+-r_-)^2}$$
$$-\frac{\pi}{6h\beta^2}\cdot m^2\Omega_0^2\cdot\frac{(r_+^2+a^2)^3}{(r_+-r_-)^2}-\frac{2}{3\pi}\frac{1}{h\beta}\cdot\zeta(1)\cdot m^3\Omega_0^3\cdot\frac{(r_+^2+a^2)^3}{(r_+-r_-)^2}. \tag{32}$$

Using the relationship between free energy and entropy

$$S=\beta^2\frac{\partial E}{\partial\beta}, \tag{33}$$

and taking the ultraviolet cutoff factor $h=\frac{T_+}{90}$ ('tHooft, 1985), the entropy becomes

$$S_1=\frac{7}{8}\frac{A_h}{4}+O(\zeta(1))+\text{finite}. \tag{34}$$

where $\zeta(n)$ is the Riemann ζ function, $A_h=4\pi(r^2+a^2)$ is the area of event horizon.

In the following, we calculate the entropy for F_2 component making use of brick-wall model and taking the same cutoff factor described above, that is, when $r\le r_++h, r\ge L\gg r_+$, it is required that

$$F_2=0. \tag{35}$$

Because the radial component $R_{+1/2}$ of F_2 satisfies Eq. (21), in the same way we may let $R_{+1/2}=e^{i\omega_2(r)}$, and then substitute it into Eq. (21), and having used WKB approximation, we have

$$K_2^2(r)=\Delta^{-1}\left[\frac{K^2}{\Delta}-l(l+1)+1\right]. \tag{36}$$

where $K_2=\partial_r W_2(r)$ is the radial wave function. From this, the free energy corresponding to F_2 is

$$\beta E_2=-\frac{\beta}{\pi}\int dl\ (2l+1)\int d\omega\left(1+e^{\beta(\omega-m\Omega_0)}\right)^{-1}\int_{r_++h}^{L}K_2\,dr\cdot \tag{37}$$

Calculating (37), we have found that there is the same expression for E_2 and E_1.

Using (33) and taking $h=\frac{T_+}{90}$, the entropy becomes

$$S_2=\frac{7}{8}\cdot\frac{A_h}{4}+O(\zeta(1))+\text{finite}. \tag{38}$$

In the same way, we can calculate the entropies of G_1 and G_2 respectively. It follows that the free energy and entropy of G_1 and G_2 are respectively equal to the ones of F_1 and F_2, so the Fermi entropy in Kerr black hole background is:

$$S = \sum_j S_j = \frac{7}{2} \cdot \frac{A_h}{4} + O(\zeta(1)) + \text{finite}. \quad (39)$$

4. DISCUSSION AND CONCLUSION

When $a = 0$, it follows that $\Omega_0 = 0$. Because there are factors of a and Ω_0 for the second and third terms in (39), so when $a = 0$, the second and third terms are all equal to zero, and only the first term $\frac{7}{2}\frac{A_h}{4}$ remains for (39). The first term $\frac{7}{2}\frac{A_h}{4}$ of (39) is just that of Fermi entropy of Schwarzschild black hole; it is equal to the Bose entropy multiplied by $\frac{7}{2}$ of Schwarzschild black hole. This conclusion is consistent with the one obtained by functional analysis in de Alwis and Ohta (1995).

Thus for the general Kerr black hole, the expression for fermionic entropy is

$$S = \frac{7}{2} \cdot \frac{A_h}{4} + O(\zeta(1)) + \text{finite}.$$

This conclusion can be extended to Kerr-Newman black hole and Kerr–Newman–Kasuya black hole.

ACKNOWLEDGMENT

This work is supported by National Nature Science Fund (No:19873013).

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